# GRAPHICAL SOLUTION OF THE PENMAN EQUATION FOR POTENTIAL EVAPOTRANSPIRATION

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## ABSTRACT

A practicable solution of the Penman formula for potential evapotranspiration by use of graphs is presented. For ease in computation, three graphs are used. These graphs, designed for Columbia, S.C., can be adapted for any location by relabeling or correcting two of the families of curves.

## 1. INTRODUCTION

Potential evapotranspiration is a measure of the maximum possible water loss from an area under a specified set of weather conditions. This phenomenon may be defined as the water loss from a vegetated surface which is supplied with adequate water at all times. Penman [1] has shown that by the simultaneous solution of a Dalton-type equation and the energy-balance equation one can obtain a fairly simple, yet reasonably sound, equation for estimating potential evapotranspiration. The general form of the Penman equation is

$$E_t=\frac{K(Q_nD+FE_a)}{D+F} \tag{1}$$

where $E_t$ is potential evapotranspiration; $D$ is slope of the saturation vapor pressure vs. temperature curve at the air temperature; $F$ is a constant (we use 0.27) in the equation $R=F\ (T_s-T_a)/(e_s-e_a)$, where $R$ is the Bowen ratio; $Q_n$ is net radiant energy exchange in the same units as the evaporation; $E_a$ is evaporation estimated by a simple equation of the Dalton type; $T_s$ is temperature of the water surface; $T_a$ is temperature of the air; $e_s$ is saturation vapor pressure at the temperature of the water surface; $e_a$ is vapor pressure of the air above the water; and $K$ is a factor applied to account for the difference between open water and a vegetated surface evaporation (Penman suggests 0.6 to 0.8 depending on the season; for the sake of simplicity, we use 0.7).

The evaluation of the Penman formula, equation (1), involves estimating or measuring the energy received by a surface. Because it is presumed, in the Penman approach, that the distribution of energy takes place according to the so-called Bowen ratio, no decision has to be made as to transfer of sensible heat to the water surface.

Rijkoort [2] and van der Bijl [3] present nomographs for the solution of the Penman equation. This paper describes a simpler graphical solution than van der Bijl's nomographs.

## 2. GRAPHICAL SOLUTION

The equation first derived by Penman [1] is used here with some modification. Units are converted so that the formula will yield estimates of potential evapotranspiration in units of inches of water. For the purpose of these computations, the latent heat of vaporization is computed for approximately the average temperature observed during the summer in South Carolina. $Q_n$ in equation (1) is given by

$$Q_n=R_a(1-r)(0.18+0.55n/N) -ST_a^4(0.56-.092\sqrt{e_a})(0.10+0.90n/N) \tag{2}$$

where $R_a$ is mean monthly extraterrestrial radiation in mm. day$^{-1}$ (evaporation equivalent); $r$ is reflection coefficient, used as 0.05 in Penman formula; $n/N$ is ratio of duration of actual sunshine to maximum possible sunshine; $S$ is Boltzman's constant$=2.01\times10^{-9}$ mm. day$^{-1}$ (evaporation equivalent); $T_a$ is air temperature in F.°; and $e_a$ is actual vapor pressure of air in mm. Hg.

$E_a$ in equation (1) is estimated from a Dalton-type equation:

$$E_a=0.35(e_s-e_a)(0.5+.0098\,u_2) \tag{3}$$

where $e_s$ is saturation vapor pressure of the air in mm. Hg.; $e_a$ is actual vapor pressure of air in mm. Hg.; $u_2$ is wind speed at 2 m. in miles per day; and $E_a$ is in units of mm. day$^{-1}$. The constant 0.5 is used in lieu of the constant 1 that Penman used in 1948.

By substitution of (2) and (3) into (1) and conversion of units so that the formula yields inches instead of millimeters of water, the Penman equation becomes [4]

$$E_t=\frac{(.7)(.0394)}{D+.27}DR_a(1-r)(.18+.55\,n/N) +(.27)(.35)(e_s-e_a)(.5+.0098\,u_2)\frac{(.7)(.0394)}{D+.27} -\frac{D(.7)(.0394)}{D+.27}ST_a^4(.56-.092\sqrt{e_a})(.10+.90\,n/N). \tag{4}$$

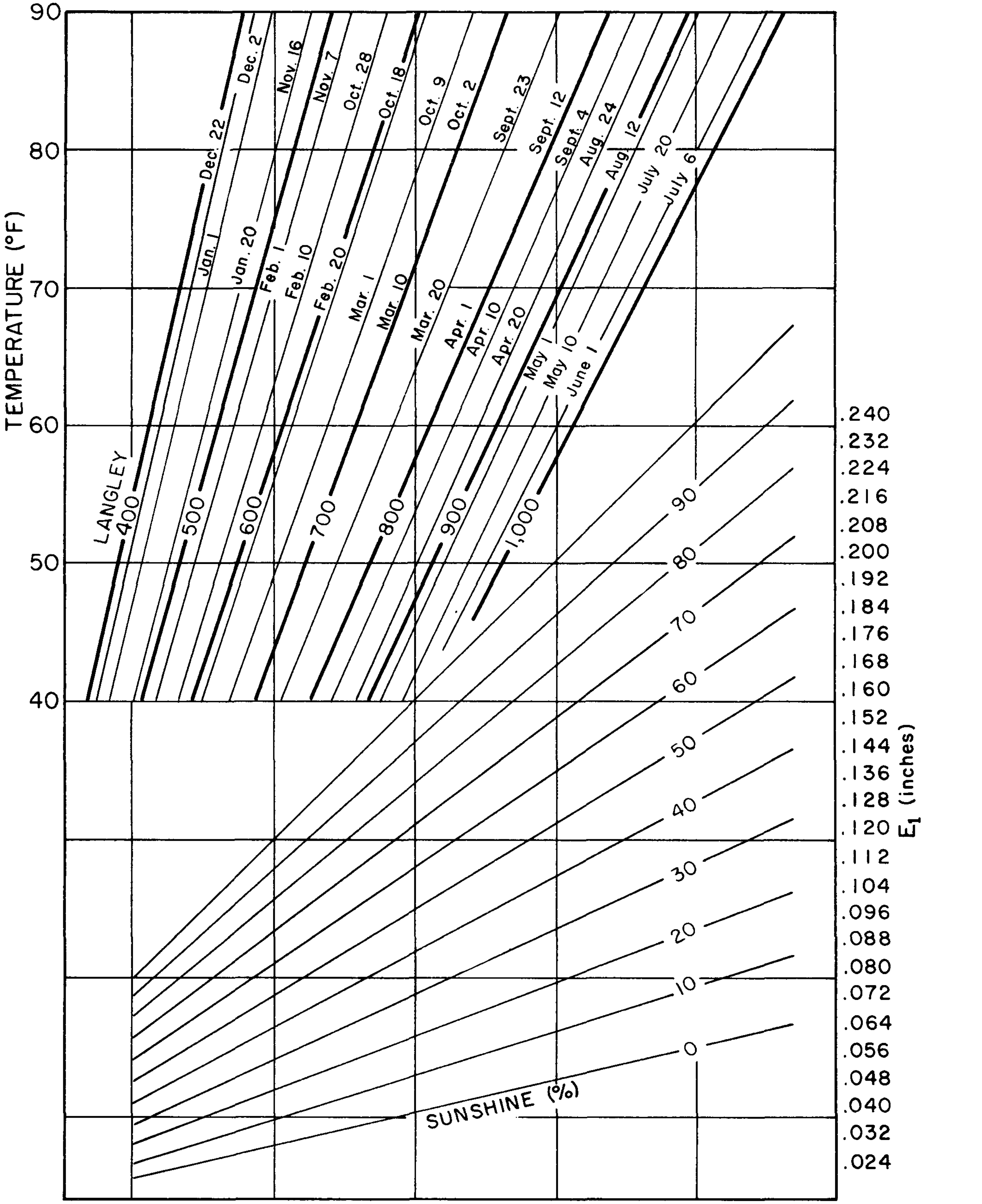


FIGURE 1.—First of three graphs for computation of potential evapotranspiration by Penman formula. This graph yields $E_1$ as given by equation (6). To use the graph, (1) enter temperature on the upper left scale, (2) move horizontally to the radiation value or calendar date, (3) descend vertically to sunshine percentage, and (4) move horizontally and read value of $E_1$ on the lower right scale. Dates shown on this graph are for Columbia, S.C.

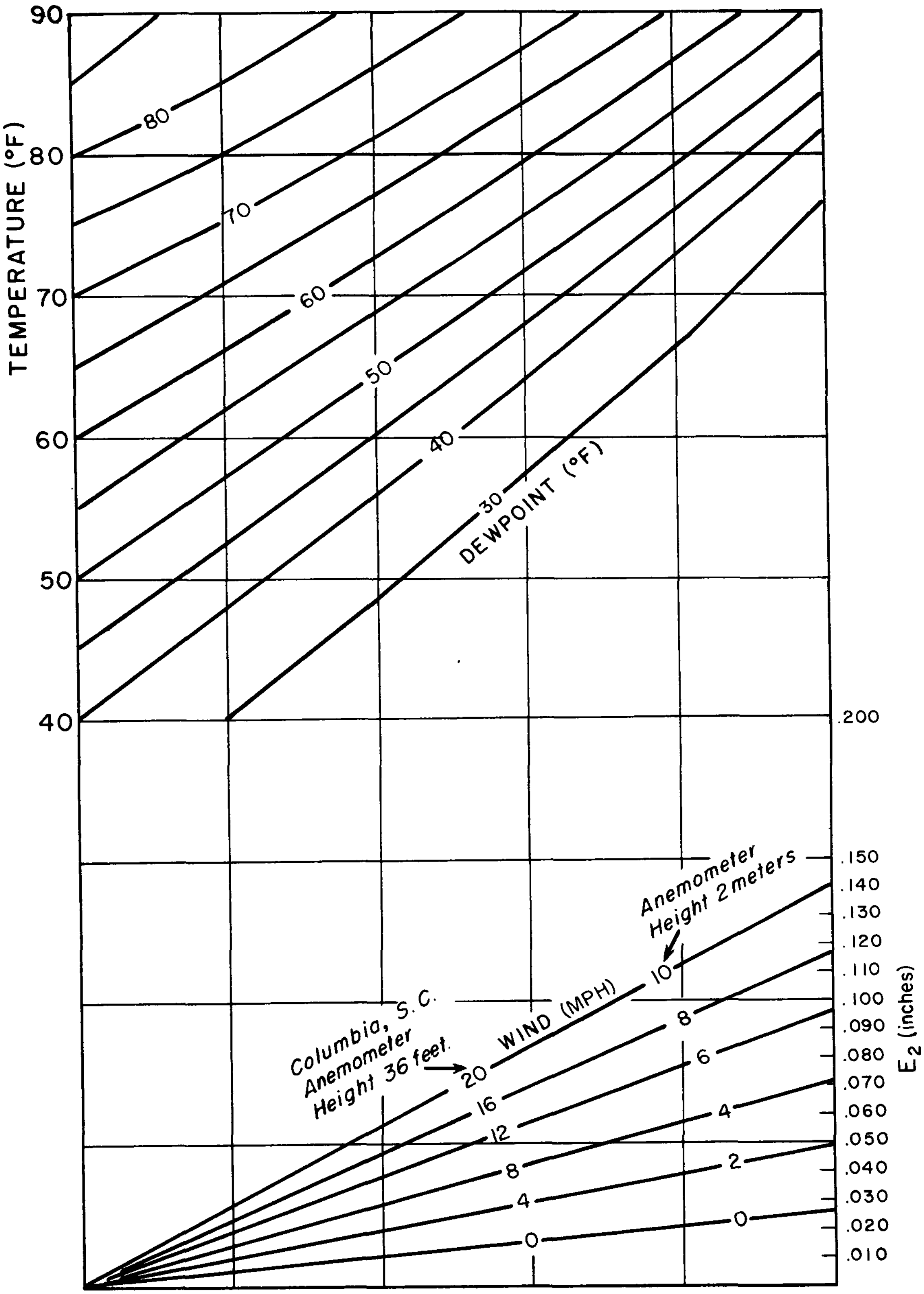


FIGURE 2.—Second of three graphs for computation of potential evapotranspiration by Penman formula. This graph yields $E_2$ as given by equation (7). To use this graph, enter temperature on the upper left scale, (2) move horizontally to the dew point (average), (3) descend vertically to the wind speed (daily average), and (4) move horizontally and read value of $E_2$ on the lower right scale. Wind speeds for the Columbia, S.C., anemometer (36-ft. height) are shown to the left of those for an anemometer at 2-m. height.

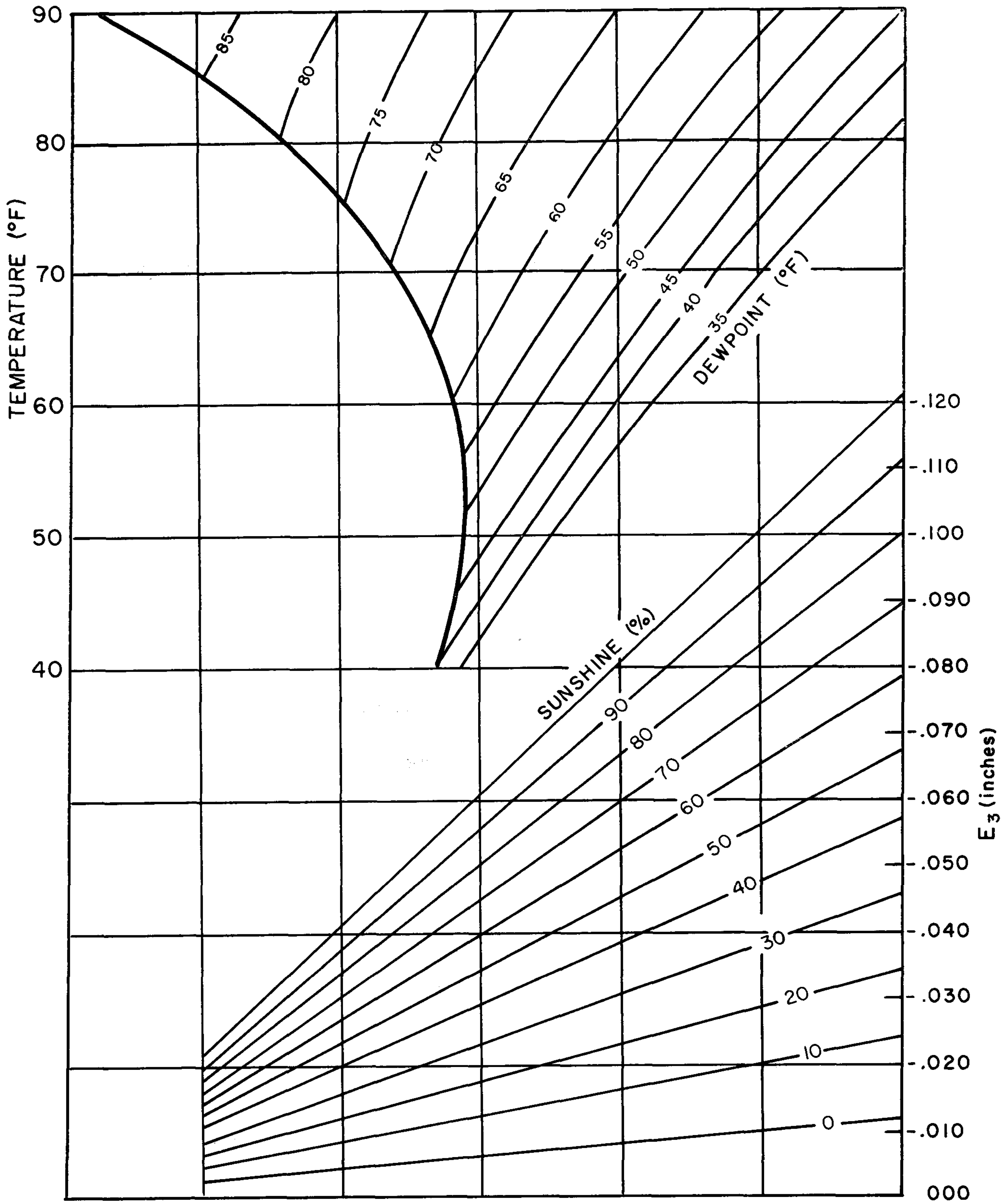


FIGURE 3.—Third of three graphs for computation of potential evapotranspiration by Penman formula. This graph yields $E_3$ as given by equation (8). To use the graph, (1) enter temperature on the upper left scale, (2) move horizontally to dew point, (3) descend vertically to sunshine percentage, and (4) move horizontally and read value of $E_3$ on the lower right scale. Note that the values of $E_3$ are negative. The computed value of potential evapotranspiration, $E_t$, is given as the algebraic sum of $E_1$, $E_2$, and $E_3$ obtained from figures 1, 2, and 3, respectively.

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A graphical solution of the Penman formula can be devised from the three terms on the right side of (4). Thus

$$E_t = E_1 + E_2 + E_3 \tag{5}$$

where

$$E_1 = \frac{(.7)(.0394)}{D+.27} DR_a(1-r)(.18+.55\,n/N) \tag{6}$$

$$E_2 = (.27)(.35)(e_s - e_a)(.5+.0098\,u_2)\frac{(.7)(.0394)}{D+.27} \tag{7}$$

$$E_3 = -(D)\frac{(.7)(.0394)}{D+.27} ST_a^4(.56-.092\sqrt{e_a})(.10+.90\,n/N). \tag{8}$$

Figure 1 is the graphical solution of equation (6). Since $D$ is a function of the air temperature, the upper part of figure 1 is drawn as temperature versus $R_a$, the extraterrestrial radiation, while the lower portion of the figure accounts for the influence of the variable $n/N$. The extraterrestrial radiation for any particular location is fixed by the date of the year. The reader may find complete tables of $R_a$ values for all latitudes in the *Smithsonian Meteorological Tables* [5]. It is possible, therefore, to adapt figure 1 to any fixed location by labeling the indicated isolines of $R_a$ with the corresponding dates for that location. To use figure 1 the reader should enter the upper left hand margin with the air temperature, move to the right until the proper $R_a$ is intersected, then descend to the $n/N$ value. The solution $E_1$ is then found on the lower right hand margin.

Figure 2 is the graphical solution of equation (7). The upper portion of this figure solves the actual and saturation vapor pressure functions. The relation at the bottom of the figure accounts for the effect of wind speed. To use figure 2, enter the upper left hand margin with the actual air temperature, move horizontally to the dewpoint, then descend to the wind speed curve. The solution $E_2$ is found on the extreme lower right hand margin.

It should be noted that the wind speed $u_2$ is that recorded at the 2-m. level. If this value is not known, a measurement at a height $h$ may be converted by use of

$$u_2 = \frac{u_h \log 6.6}{\log h} \text{ or } u_h = \frac{u_2 \log h}{\log 6.6}$$

where the height of measurement is in feet. Thus this figure can be adapted to any particular site by relabeling the wind speeds with the corresponding values for the particular anemometer height; e.g., for 36 ft. as applicable to the Columbia, S.C., anemometer, given by the second set of wind speed labels.

Figure 3 is the graphical solution of equation (8). The upper portion of this figure solves the temperature-vapor pressure relation. The lower part of the graph accounts for the influence of the parameter $n/N$. To use figure 3, enter the upper left hand margin with the air temperature, move horizontally to the dew point, then descend vertically to the percentage of sunshine. The answer $E_3$ is found on the lower right hand margin.

The solution $E_t$ to equation (4) is then accomplished by adding results $E_1$, $E_2$, and $E_3$ found from figures 1, 2, and 3. For example, at Columbia, S.C., on May 1, with average temperature 80° F., dew point 70° F., wind speed 8 m.p.h., and sunshine 10 percent, figure 1 gives $E_1=0.072$ in.; figure 2 gives $E_2=0.025$ in.; and figure 3 gives $E_3=-0.011$. Thus, from equation (5)

$$E_t = 0.072 + 0.025 - 0.011 = 0.086 \text{ in.}$$

## 3. CONCLUSION

This paper presents a graphical solution of the Penman formula as adapted for Columbia, S.C. The graphical solution is given in the form of three graphs (figs. 1, 2, 3) which may be adapted for any location by changing figures 1 and 2 as described in the text.

## ACKNOWLEDGMENT

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## REFERENCES

1. H. L. Penman, "Natural Evaporation from Open Water, Bare Soil, and Grass," *Proceedings of the Royal Society of London*, Series A, vol. 193, 1948, pp. 120–145.
2. P. J. Rijkoort, "A Nomogram for the Determination of Pan Evaporation Adapted from the Penman Method," *Report 11–143* Royal Netherlands Meteorological Institute, 1954.
3. W. van der Bijl, "Evapotranspiration Problem: First Contribution," Kansas State College, Dept. of Physics, *Report 1956–1957* on Contract Cwb–8806, 94 pp.
4. N. Kronberg and J. C. Purvis, Potential Evapotranspiration Study, unpublished, 1957.
5. R. List (ed.), *Smithsonian Meteorological Tables*, 6th Rev. Edition, Smithsonian Institution, Washington, D.C., 1951.